Frequency Doubling Using Ferrite Slabs, Particularly Planar Ferrites*

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Summary—**An analysis is given of the results of frequency doubling experiments using relatively thick ferrite slabs. The primary frequency was 9100 Mc. It is shown that the dielectric properties of the ferrite cause the double-frequency wave to behave as a surface wave. This tends to concentrate the double-frequency fields in the vicinity of the ferrite, thus increasing its interaction with the oscillating magnetic-dipole moments that generate these fields and increasing the conversion efficiency. However, the surface-wave effect also causes a phase difference between the magnetic-dipole moments and the double-frequency fields, and this limits the efficiency obtainable through this effect. Experimental data give reasonable agreement with calculated results. Spinwave effects are considered, and it is shown that they can contribute to harmonic generation if the precession orbits of their magnetization vectors are highly elliptical. The best conversion efficiency obtained in these experiments was −11.1 db at a peak input power level of 6300 w. Planar ferrites have inherent advantages over isotropic ferrites because the planar anisotropy greatly increases the ellipticity of the precession of the magnetization vectors of both the uniform mode and the spinwaves. The relatively large dielectric constant of planar ferrites is also helpful to frequency doubling.**

Introduction

Auld and his co-workers[1] have developed a reasonably accurate theory for frequency doubling for the case of a very thin isotropic ferrite slab. However, the conversion efficiencies have been very poor for this geometry. A high degree of efficiency in frequency doubling has been obtained using relatively thick samples. Conversion losses of only 5 and 6 db have been reported[2,3] in frequency doubling from 9 to 18 kMc, with a peak power at the primary frequency of approximately 30 kw. However, these results have been obtained by purely empirical methods. There has been no explanation to account for these high efficiencies. It is the purpose of this paper to present an analysis of some of the factors responsible for the high efficiency obtainable with thick ferrite slabs. At the high powers required for efficient frequency doubling, spinwaves are excited. This paper will also discuss some spinwave effects observed in the experiments.

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[1] B. A. Auld, H. J. Shaw, and D. K. Winslow, "Traveling-wave frequency doubling in ferrites," *J. Appl. Phys.*, suppl. to vol. 32, pp. 317S–318S; March, 1961.

[2] S. L. Melchor, W. P. Ayres, and P. H. Vartanian, "Microwave frequency doubling from 9 to 18 kMc in ferrites," Proc. IRE, vol. 45, pp. 643–646; May, 1957.

[3] H. J. Shaw, D. K. Winslow, and E. K. Kirchner, "Ferrite Doubler," W. W. Hansen Labs. of Physics, Stanford Univ., Stanford, Calif., Quart. Status Rept. No. 4, Contract Nonr 225 (48); March, 1960.

Experimental Setup, Ferrite Properties

All experiments in this paper were performed at a primary frequency of 9100 Mc unless otherwise indicated. The pulse duration was 0.5 μsec and the pulse-repetition rate (PRR) 200 pps. None of the samples heated up at the power levels used. The waveguide and sample geometries are shown in Fig. 1. Dimensions c and d were 0.4 and 0.8 in, respectively. Dimension b was always 0.100 in. The sample length was sufficiently long so that substantially all the primary frequency energy was absorbed by the ferrite at resonance. When planar ferrites were used, the easy plane of magnetization was in the XY plane. This orientation permitted both the biasing field and the RF magnetic field to be in the easy plane.

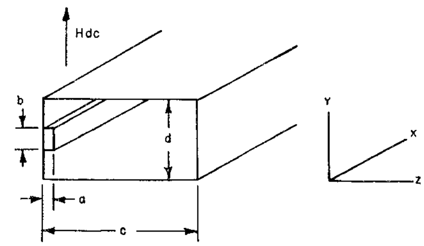

Fig. 1—Waveguide and sample geometry.

The experimental setup used is shown in Fig. 2. For all experiments except the experiment for maximum conversion efficiency, a resistance card was placed parallel to the broad face of the rectangular waveguide on the generator side of the sample. This did not affect the primary frequency wave, but acted as an attenuator for double-frequency energy propagating towards the generator. In the experiments to obtain maximum conversion efficiency, the resistance card was replaced by a phase shifter and constricted waveguide.[4] The phase shifter was a thin polystyrene sheet placed parallel to the broad face of the waveguide. It did not affect the primary frequency wave, but shifted the phase of the double-frequency wave. The constriction in the waveguide did not affect the primary-frequency wave, but totally reflected the double-frequency wave.

The properties of materials referred to in this paper are shown in Table I.

[4] Melchor, *et al.*, *op. cit.*, Fig. 1.

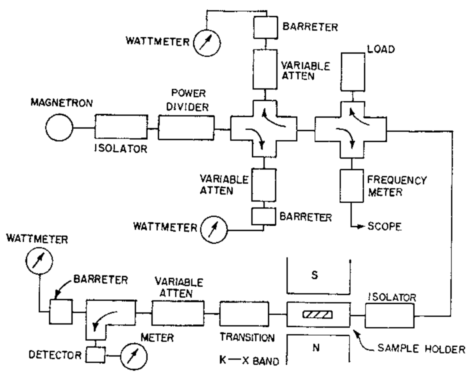

Fig. 2—Test setup.

TABLE I

MATERIAL PROPERTIES

Desig-nation	$4\pi Ms$ gauss	Ha oe	ΔH oe	ϵ_r	Composition
P1	2480	9200	410	18	$Ni_{0.6}\ Cu_{0.6}\ Zn_{0.8}\ Y$
P2	2130	7500	510	25	$Ni_{0.4}\ Cu_{0.4}\ Zn_{1.2}\ Y$
P3	2500	12,000	260	19	$Cu_{0.4}\ Co_{0.4}\ Zn_{1.2}\ Y$
P4	2500	12,000	290	18	$Cu_{0.4}\ Co_{0.4}\ Zn_{1.2}\ Y$
P5	2270	6900	338	18	$Cu_{0.5}\ Zn_{1.5}\ Y$
Y	1800	—	70	15	Yttrium iron garnet
S	4500	—	165	12	NiZn ferrite

Linewidth was measured on a 0.01-in slab for materials Y and S, and on an 0.02-in slab for the planar ferrites. This measurement was made at very low power levels. Ha is the magnetic anisotropy field of the planar ferrites. The dielectric constant of all materials except Y was measured at 7000 Mc. The value of the dielectric constant for material Y was taken from the literature. Material S is the Trans-Tech material TT2-111.

EQUATION FOR DOUBLE-FREQUENCY OUTPUT OF VERY THIN FERRITE SLABS

The case of the very thin ferrite slab will be considered first, since it will be used in the development of the theory for the thicker slab. The double-frequency peak power output for a very thin ferrite slab is given in (1).

$$P_{2\omega} = \frac{0.00417}{cd}\left[\frac{P_\omega}{\Delta H}\right]^2 \frac{1}{1+\left(\frac{2H_i}{\delta}\right)^2}\,\frac{\lambda_0}{\lambda_g} \qquad (1)$$

$$H_i = \frac{\omega}{\gamma}$$

$$\delta = Ha + 4\pi M_s(N_z - N_X).$$

The other terms are commonly used and assumed known. Practical cgs units have been used. It should be noted that due to assumption 4) below, the ratio λ_0/λ_g is the same for the primary frequency as for the double frequency. This equation is similar to one derived by Auld, *et al.*[1] However, Auld's equation is applicable only to isotropic ferrites. The above equation includes planar ferrites. The steps in the derivation of (1) are outlined in the Appendix.

It should be noted that the double-frequency transverse fields are orthogonal to the primary frequency transverse fields, and two modes are possible; the TE_{01} and TE_{02} modes. Only the TE_{01} mode need be considered, since only this mode will couple into the K band waveguide.

The following assumptions were made in deriving (1): 1) The ferrite slab is thin enough so that perturbation theory is applicable; 2) The ferrite slab is very long; 3) $b \ll d$; 4) $c/d = 2$ (Auld and Omori[5] have noted that, under this condition, the phase velocity of the primary frequency and the TE_{01} mode of the second harmonic are equal, so that a traveling-wave effect is present); 5) The applied power is sufficiently low so that spinwave effects do not occur; 6) Losses due to ferromagnetic resonance greatly exceed the energy radiated into the second harmonic; and 7) The biasing field is adjusted for ferromagnetic resonance, *i.e.*, maximum absorption of energy.

The term δ in (1) represents the asymmetry in the plane perpendicular to the biasing field, due to the magnetic anisotropy field and the demagnetizing factors. This asymmetry is essential for double-frequency generation, since if there is complete symmetry in the perpendicular plane the magnetization vectors will precess in pure circular orbits. The longitudinal magnetization will then not vary with time, and there will be no double-frequency output. An asymmetry in the perpendicular plane causes the magnetization vector to precess in an elliptical orbit. The longitudinal magnetization will now vary at double the input frequency, and efficient double-frequency generation is possible. The larger the asymmetry, the greater the ellipticity of the orbit, and the greater the conversion efficiency. The advantage of planar ferrites with large values of magnetic anisotropy fields is evident.

ANALYSIS OF THICK SLABS

Introduction, Surface-Wave Effect

According to (1), the double-frequency power output should be independent of the ferrite cross-section area. However, it is observed experimentally that as the thickness of the ferrite slab is increased (with height remaining constant), the efficiency of double-frequency output initially increases, reaches a peak, and then decreases. The critical thickness at which the peak output occurs varies from material to material. It is the purpose of this section to present a theory explaining this behavior.

[5] B. A. Auld and N. Omori, "Ferrite Non-Linear Propagation," W. W. Hansen Labs. of Physics, Stanford Univ., Stanford, Calif., Quart. Status Rept. No. 4, Contract Nonr 225 (48); March, 1960.

Eq. (1) shows that the double-frequency power is inversely proportional to the waveguide cross section. Examination of the steps involved in the derivation of (1) shows that m_t, the total effective double-frequency magnetic-dipole moment, is independent of the cross-section area. The dependence on the cross-section area arises in the coupling between the double-frequency magnetic-dipole moment and the double-frequency electromagnetic field it generates. If the double-frequency electromagnetic field is spread out over a large area, the interaction between the dipole moments and the field is relatively weak; *i.e.*, the radiation resistance is low, and conversion efficiency is low. If the double-frequency electromagnetic field is confined to an area close to the ferrite, the radiation resistance and hence the conversion efficiency is increased.

We will now show that due to its dielectric properties, the ferrite will tend to concentrate the double-frequency fields in its vicinity, the degree of the concentration depending on the dielectric constant and thickness of the ferrite slab; *i.e.*, the double-frequency fields constitute a surface wave.

Derivation of Field Equations

The model that will be used in making numerical calculations will be the parallel-plane transmission line with a dielectric medium against one plane as shown in Fig. 3. The effects of some of the differences between the model and the actual geometry is discussed in the Appendix. The matter of coupling between the surface wave and the normal rectangular waveguide modes is also considered in the Appendix.

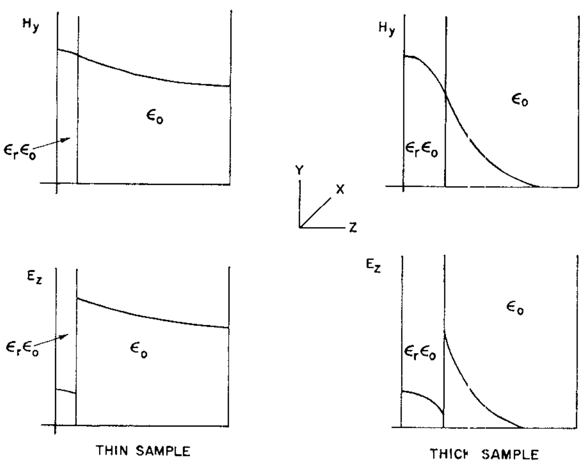

Fig. 3—Dielectrically loaded parallel-plane waveguide.

The only mode of the parallel-plane waveguide that will be considered is the lowest order mode, and this is a TM mode, with the RF magnetic field in the Y direction. Since the biasing field is also in the Y direction, the ferrite is saturated in that direction, and the relative permeability can be taken as one. A transverse resonance technique can be used to determine the fields for this model. This leads to

$$\kappa_\epsilon \tan \kappa_\epsilon a = \epsilon_r \kappa_0 \tan \kappa_0 (c - a). \qquad (2)$$

ϵ_r is the relative dielectric constant of the ferrite. κ_ϵ is the phase constant in the dielectric, and κ_0 is the attenuation constant in the empty space. (The phase constant is zero in the empty space.) From the relations,

$$k^2 = k_x^2 - \kappa_0^2 \qquad (3a)$$

$$\epsilon_r k^2 = k_x^2 + \kappa_\epsilon^2 \qquad (3b)$$

we obtain

$$\kappa_\epsilon^2 + \kappa_0^2 = k^2(\epsilon_r - 1). \qquad (4)$$

$k = 2\pi/\lambda_0$ is the phase constant in free space. k_x is the phase constant of the mode in the X direction. Eqs. (2) and (4) lead to a solution of the problem.

The transverse components, which are the only components of interest for this analysis, are given in (5).

In dielectric:

$$H_y = \cos \kappa_\epsilon z$$

$$E_z = \frac{k_x}{\omega \epsilon_0 \epsilon_r} H_y.$$

In air:

$$H_y = \frac{\cos \kappa_\epsilon a}{1 + e^{-2\kappa_0(c-a)}} \left[e^{-\kappa_0(z-a)} + e^{\kappa_0(z-a) - 2\kappa_0(c-a)} \right]$$

$$E_z = \frac{k_x}{\omega \epsilon_0} H_y. \qquad (5)$$

A sketch of the transverse components for a thin sample and a thick sample are shown in Fig. 3.

Gain in Conversion Efficiency Due to Concentration of Double-Frequency Fields

As we have noted before, the effect of the surface wave is to concentrate the double-frequency fields generated by the oscillating magnetic-dipole moments, into the vicinity of the magnetic-dipole moments, thus increasing the effective radiation resistance of the dipoles and the efficiency of the double-frequency conversion. The actual increase in efficiency will now be calculated. It will be assumed in the general development that the primary frequency field is not disturbed by increasing the slab thickness, and that the effective linewidth of the samples remain unchanged as thickness is increased.

Consider that we have a magnetic-dipole moment and that it excites a wave in a uniform waveguide. From Goubau[6] we can derive an expression, shown in

[6] G. Goubau, "On the excitation of surface waves," PROC. IRE, vol. 40, pp. 865–868; July, 1952.

(6), giving the magnitude of the excited magnetic field, h, at the location of the dipole moment m

$$h = \frac{\omega_{\text{dipole}} m}{2} G. \tag{6}$$

G is determined as follows: Assume some magnetic field amplitude at the location of the magnetic-dipole moment. For the particular mode under consideration, calculate the corresponding power flowing through the waveguide. (For the model we use, in which the waveguide is unlimited in the Y direction and has no variation in the Y direction, power per unit length in the Y direction should be calculated.) G is equal to the square of the assumed magnetic field divided by the power.

The power in the excited wave is clearly equal to h^2/G. From (6), it therefore follows that:

$$\text{Power} = \left(\frac{\omega_{\text{dipole}} m}{2}\right)^2 G. \tag{7}$$

Thus, the efficiency of the conversion is proportional to G.

The value of G for the parallel-plane waveguide, as a function of the thickness of the dielectric, can be readily calculated. The increase in efficiency due to the concentration of the fields in the vicinity of the ferrite for a given thickness is equal to the ratio of the value of G for that thickness $G(a)$, to the value of G in the case of a vanishingly small thickness, $G(0)$. This ratio will be designated as the normalized G factor. A plot of the normalized G factor as a function of thickness is shown in Fig. 4.

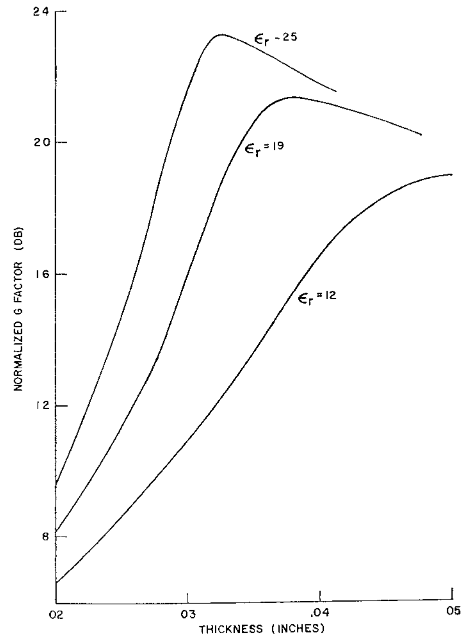

Fig. 4—Variation of normalized G factor with thickness.

Change in Phase Constant Due to Surface Wave Effect, and its Effect on Conversion Efficiency

For vanishingly thin slabs, the phase velocities of both the primary frequency wave and the double-frequency wave in the parallel-plane model we are discussing will be the same. Therefore, the double-frequency wave will be in synchronism with the double-frequency magnetic-dipole moments exciting it at all points. As the slab thickness is increased, the surface wave effect causes the phase velocities of both the waves to decrease. However, the phase velocity of the double-frequency wave will decrease at a far faster rate than the primary frequency wave. The difference in the phase velocities will cause the double-frequency wave to depart from synchronism with its exciting source and thus reduce the conversion efficiency. In the following analysis, we will make the approximation that the phase velocity of the primary wave is unaffected by the thickness of the ferrite slab.

We need first to determine the ratio R of the phase constant of the double-frequency wave at a given ferrite slab thickness to the phase constant in the case of a vanishingly thin slab. This can readily be determined from (3a) and (3b).

$$R = \sqrt{\frac{\kappa_\epsilon^2 + \epsilon\kappa_0^2}{\kappa_\epsilon^2 + \kappa_0^2}}\,. \tag{8}$$

A plot of R as a function of thickness is shown in Fig. 5. The decrease in conversion efficiency is given by [7]

$$D = \left| \frac{\int_0^\infty e^{-[2\alpha + j\beta_2(R-1)]x}dx}{\int_0^\infty e^{-2\alpha x}dx} \right| = \frac{1}{1 + \left[\frac{\beta_2}{\alpha}(R-1)\right]^2}\,. \tag{9}$$

β_2 is the phase constant of the double-frequency wave in the rectangular waveguide for the case of a vanishingly small slab. The use of β_2 in (9) is discussed in the Appendix. It should be noted that the value of α is proportional to thickness.

The over-all gain due to the surface-wave effect can be obtained by multiplying the normalized G factor by D. The results are plotted in Fig. 6. The attenuation indicated in Fig. 6 is the attenuation at resonance of the primary frequency wave at a thickness of 0.04 in. Fig. 6 shows that the over-all gain (for a given level of attenuation) is approximately proportional to the square root of the dielectric constant, and the thickness at which the maximum over-all gain occurs is approximately inversely proportional to the square root of the dielectric constant.

[7] Auld, *et al.*, *op. cit.* (footnote 1), see (2).

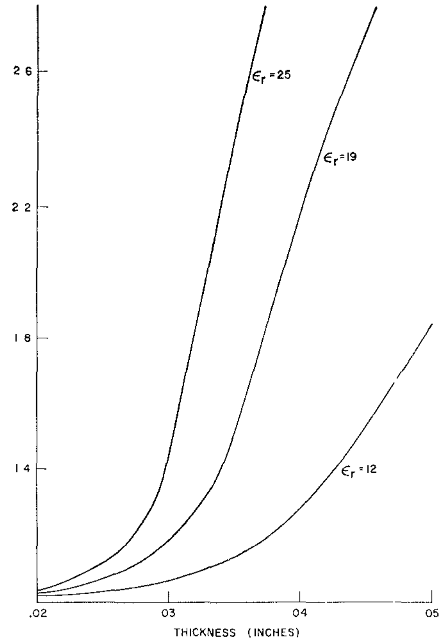

Fig. 5—Variation of R with thickness.

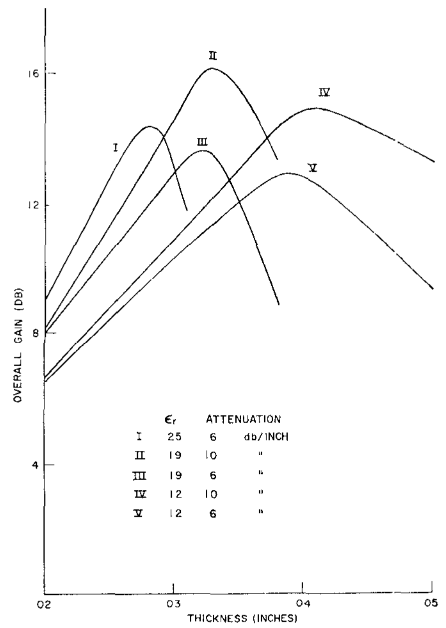

Fig. 6—Variation of over-all gain with thickness.

Experimental Results

In this section, the experimental data will be given and compared to theoretically calculated results. The basis of comparison will be the maximum over-all gain and the thickness at which the maximum over-all gain occurred. The experimental over-all gain was calculated by taking the ratio of the measured power at a given thickness to the power calculated from (1). The input power in all cases was 6300 w peak.

Before proceeding to the experimental data on over-all gain, some comment on the variation of attenuation with thickness is necessary. From the assumption that the primary frequency wave was not perturbed by the ferrite slab and that the effective linewidth was the same at all ferrite slab thicknesses, it followed that the attenuation per unit length was proportional to slab thickness. Some of the ferrites behaved in this manner, but others did not. The variation of attenuation per unit length with thickness of several of the ferrites used is shown in Table II.

TABLE II
RELATIVE ATTENUATION OF FERRITES

Material	Thickness (Inches)		
	0.02	0.04 (laminar)	0.04 (solid)
S	1.00	0.81	0.78
P1	1.00	1.07	—
P2	1.00	1.00	1.00
P3	1.00	1.25	2.00
P5	1.00	1.26	—

Relative attenuation was calculated from

$$\alpha_r = \frac{\alpha(a)}{\alpha(0.02)} \times \frac{0.02}{a}, \tag{10}$$

where $\alpha(a)$ and $\alpha(0.02)$ were the attenuations per unit length at thicknesses a and 0.02 respectively. In the 0.04 (laminar) configuration, the 0.04-in thickness was made up of two 0.02-in slabs one on top of the other. In the 0.04 (solid) configuration, the slab was a single piece in cross section. All measurements were made at very low power levels.

The frequency-doubling efficiency of a material was strongly influenced by the manner in which its attenuation varied with thickness, as can be seen in the following discussion.

Attenuation Proportional to Thickness: From Table II, we note that the attenuations of materials P1 and P2 are substantially proportional to thickness. A plot of the over-all gain of these materials is shown in Fig. 7. The attenuation of both these materials was very close to 6 db/in at a thickness of 0.04 in.

Comparison of Figs. 6 and 7 show that the measured maximum over-all gain is within one db of the theoretically calculated gain for both materials. The thickness at which maximum over-all gain occurred is within approximately 20 per cent of the theoretically calculated thickness for both materials.

For the relatively small thicknesses, *i.e.*, 0.03 in and less, the slope of the double-frequency output power vs the primary frequency input power (with power expressed in db) was greater than 2.0 over a portion of the range of input power. Eq. (1) implies a constant slope of 2.0. Eq. (1) was of course derived for the case of a very thin slab. However, the extension to the case of a thick slab does not change this. The presence of a slope greater than 2.0 is believed to be due to spinwaves

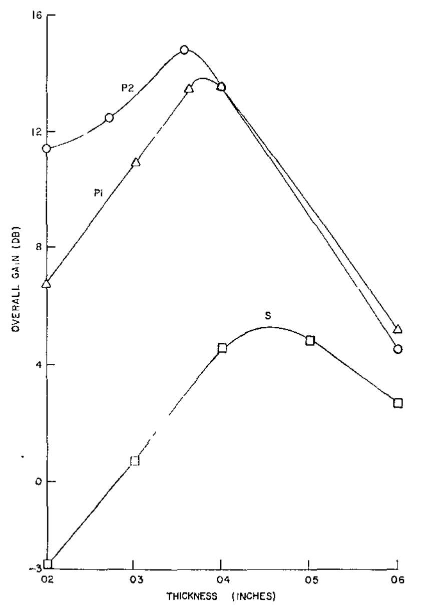

Fig. 7—Over-all gain of materials S, P1, P2.

and is discussed again in a later portion of this paper. If the slope had remained at 2.0, the gains indicated in Fig. 7 would be smaller by approximately 3–4.5 db at thicknesses of 0.03 in or less. At thicknesses at which the maximum over-all gain is achieved, and at larger thicknesses, the slope did not appreciably exceed 2.0.

Attenuation Varies Less than the First Power of Thickness: Table II shows that material S falls in this heading. Additional measurements show that the relative attenuation at a thickness of 0.01 in was 1.69, and the relative attenuation at a thickness of 0.06 in was 0.68. The decrease in relative attenuation as the thickness is increased is due primarily to the fact that the slab departs more and more from the theoretical ellipsoidal shape as the thickness is increased, so that the internal fields deviate increasingly from uniformity. This causes the effective linewidth to broaden and thus reduces the attenuation. This effect is much more pronounced with material S than with the planar ferrites tested, because its saturation magnetization is considerably higher, and its linewidth at a small thickness is considerably narrower as compared to the planar ferrites tested.

The attenuation of material S, at the thickness where the maximum gain occurred, was approximately 10 db/in. The maximum measured over-all gain of 4.8 db was considerably less than the theoretically calculated value of 15 db shown in Fig. 6. However, in calculating the measured over-all gain the linewidth measured at a thickness of 0.01 in was used. The linewidth at the thickness at which maximum gain occurred was larger than the linewidth at 0.01 in by a factor of 2.2. If the larger linewidth were used in the calculation, the theoretical over-all gain would be 11.7 db. The discrepancy between the measured and theoretical values is thus greatly reduced.

The thickness at which the maximum gain occurred is within approximately 10 per cent of the theoretical value.

Spinwaves were much in evidence in material *S*, as discussed in a later portion of this paper. However, the slope of double-frequency output power vs input power (with power expressed in db) remained very close to 2.0 up to the power level used in this experiment. The presence of spinwaves can therefore be disregarded in comparing measured and theoretical values of over-all gain.

Attenuation Varies More than the First Power of Thickness: Table II includes two materials whose relative attenuation in the laminar arrangement substantially exceeded 1. Moreover, when one of these materials was measured in the solid arrangement, its relative attenuation increased to 2. A relative attenuation greater than 1 means either that the effective linewidth decreased as thickness increased, and/or that the RF magnetic field in the ferrite increased beyond the value present in the case of a thin slab. This, of course, deviates from the assumptions used in developing the theory of the thick slab. Let us for the present disregard these deviations, but return to them later.

A plot of over-all gain and relative attenuation of a material whose attenuation varied substantially more than the first power of thickness is shown in Fig. 8. The material P4 is of the same composition as material P3, but was fired under slightly different conditions. The resultant properties are therefore slightly different. This material was cut to 0.055-in slabs one inch in length and ground down in steps to a thickness of 0.019 in. The double-frequency power output and the relative attenuation were measured at each step. Relative attenuation α_r was calculated from

$$\alpha_r = \frac{\alpha(a)}{\alpha(0.019)} \times \frac{0.019}{a}. \qquad (11)$$

$\alpha(a)$ and $\alpha(0.019)$ were the measured attenuations per unit length at thicknesses a and 0.019 in, respectively. $\alpha(0.019)$ was 4.5 db/in. An adequate number of slabs were used at each thickness to ensure that most of the input energy was absorbed by the ferrite.

Fig. 8 also shows the theoretically calculated over-all gain for this material. The value of α used at each thickness was determined from the measured attenuation at that thickness. The value of linewidth used was the value of linewdith measured with the 0.019-in slab. We note that the thickness at which the maximum efficiency occurs is within approximately 10 per cent of the calculated value. However, the maximum measured over-all gain is off by over 10 db from the theoretically calculated gain.

A large part of this discrepancy is due to the fact that the effective linewidth at the thickness where the maximum over-all gain is obtained is considerably smaller than the linewidth of the thin sample. The effective linewidth, as determined from the "half attenuation" points on a plot of attenuation vs biasing field, was

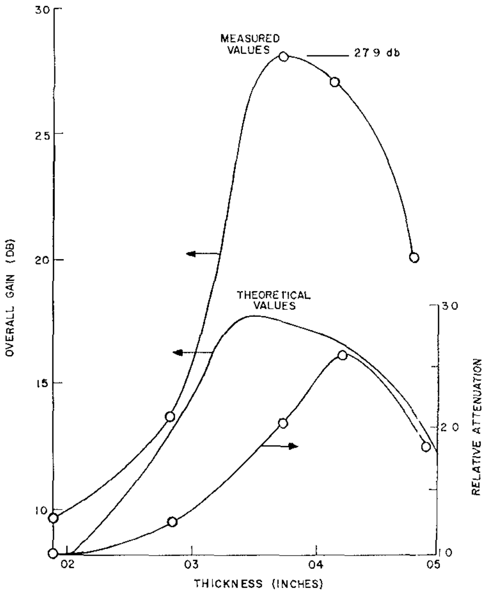

Fig. 8—Over-all gain and relative attenuation of material P4.

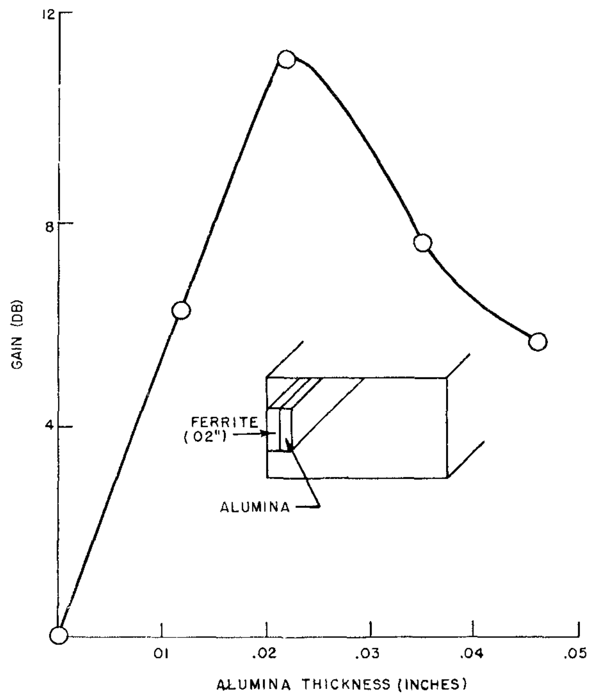

Fig. 9—Increase in conversion efficiency due to dielectric loading.

130 oe at a thickness of 0.0375 in, and 290 oe at a thickness of 0.019 in. The theoretical curve was calculated on the basis of a linewidth of 290 oe at all thicknesses. If the effective linewidth of 130 oe were used in calculating the theoretical gain at 0.0375 in, the theoretical gain would be increased by 7 db and the discrepancy between theoretical gain and measured gain would thus be reduced to approximately 3.5 db.

The mechanism responsible for the decrease in the effective linewidth and the related increase in relative attenuation is not clearly understood. It appears to be related to the linewidth of the material. It may be noted from Table II that the large relative attenuations occurred for planar ferrites with the relatively narrow linewidths, as measured at 0.02 in (see Table I). The relative attenuations of the planar ferrites with the broader linewidths were close to 1.0. Seidel[8,9] has shown that a ferrite slab with a narrow linewidth may behave in a substantially different manner than a similar ferrite slabe with a broad linewidth. It should be noted, though, that Seidel discussed full height slabs.

Table II shows that the solid arrangement of material P3 had a substantially larger attenuation than the laminar arrangement. This indicates that the presence of an interface had a strong effect in suppressing the above mechanism. The two 0.02-in pieces were held together with two small drops of Duco cement and a small air gap could be noticed. The significance of an interface was clearly demonstrated by deliberately placing a 1-mil air gap between the solid slab of material P3 and the waveguide wall to which it was fastened. The relative attenuation dropped from 2.0 to 1.46. An air gap of 5 mils reduced the relative attenuation to 1.22. The strong effect of interfaces on the behavior of ferrite slabs at resonance has also been noted by Seidel.[9]

Dielectric Loading: Since the surface-wave effects are due to the dielectric properties of the ferrite, and not its magnetic properties, it should be possible to obtain these effects with a thin ferrite slab by dielectrically loading it. A 0.02-in slab of material P3 was therefore dielectrically loaded with successive thin layers of an alumina body, and the double-frequency output determined as a function of dielectric thickness. The results are shown in Fig. 9. It is evident that the shape of the curve is similar to those in Fig. 6. The relative dielectric constant of the alumina body was 8.2, and the thickness of each layer was 0.0115 in.

Spinwave Effects

At the high peak powers required for efficient frequency doubling, spinwaves will very likely be excited. Spinwave effects are clearly manifest in the normalized susceptibility curves of materials Y and S, as shown in Fig. 10. Since susceptibility is proportional to attenuation, the normalized susceptibility curves were determined from the ratio of attenuation at a particular power level to the attenuation at very low power levels. A small length of a 0.01-in slab was used. The data in Fig. 10 were taken at 8500 Mc.

Fig. 10 also shows the double-frequency output of these materials as a function of input power. In the case of material Y, we note that the double-frequency output starts to saturate at power levels a little higher than the power level at which the normalized attenuation goes

[8] H. Seidel, "Ferrite slabs in transverse electric mode waveguides," *J. Appl. Phys.*, vol. 28, pp. 218–226; February, 1957.

[9] H. Seidel, "Viewpoints on resonance in ideal ferrite slab loaded rectangular waveguides," 1957 IRE WESCON Convention Record, pt. 1, pp. 58–69.

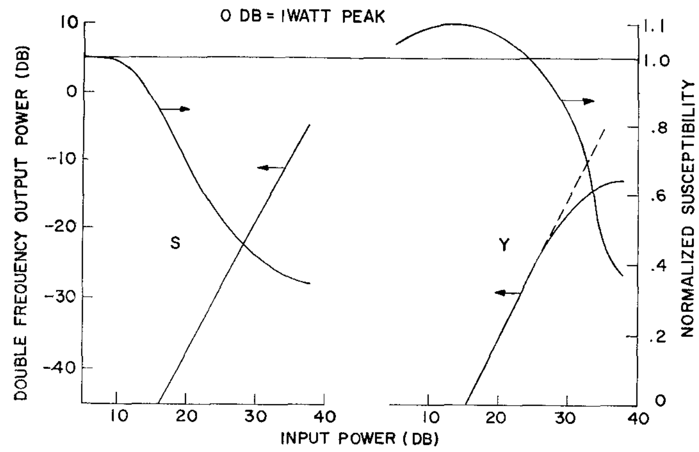

Fig. 10—Output and normalized susceptibility of materials S and Y.

below one. However, material S shows no sign of such saturation even at power levels where the normalized attenuation has dropped to less than 0.4. This behavior can be understood on the basis of Schlömann's[10,11] analysis of spinwaves as discussed below. However, it is first necessary to consider some of the effects of spinwaves on frequency doubling generally. It will be assumed that only the "second order" process is involved, in which the spinwaves have the same frequency as the uniform precession.

A basic requirement that spinwaves must meet in order to contribute to frequency doubling is that the magnetization vector precess in an elliptical orbit. The importance of the ellipticity of the orbit is discussed in the analysis of the thin slab. In isotropic ferrites the ellipticity of the orbit of the spinwave magnetization vector will depend on its direction of propagation. If the spinwave propagates parallel to the biasing field, its magnetization vector will precess in a circular orbit. If the spinwave propagates perpendicular to the biasing field, the ellipticity of the orbit will equal the ellipticity of the main precession obtained with a thin slab. Intermediate directions will have intermediate ellipticities. In the case of planar ferrites with relatively large magnetic anisotropy fields, the magnetization vectors of all spinwaves will precess in orbits whose ellipticity is comparable to that of the main precession. This is because the planar anisotropy influences the spinwaves in the same way it does the uniform precession.

In homogeneous materials, such as a good single crystal, there is a sharply defined critical field below which spinwaves will not be excited beyond their thermal level. However, in polycrystalline ferrites, there is no such sharply defined critical field. Rather, spinwaves can be excited above their thermal level at low levels of RF magnetic field, and can become very large even below the theoretically calculated critical level. Schlömann[10,11] has shown that this behavior is due to inhomogeneities that are inevitably present in polycrystalline materials due to pores, random orientation of grains, and other factors. These inhomogeneities cause scattering from the uniform mode that is driven by the RF magnetic field, to spinwaves that have the same frequency. The initial excitation is amplified by a time-varying coupling to the uniform precession in much the same way that a parametric amplifier operates. The amount of amplification and thus the amplitude of the spinwaves depend on the amplitude of the RF magnetic field.

Schlömann has also shown that it is possible to correlate the curve of normalized susceptibility vs power with the direction of propagation of the spinwaves. If the normalized susceptibility curve has an initial positive slope followed by a negative slope, it is a clear indication that most of the spinwaves are propagating substantially parallel to the biasing field. However, if the normalized susceptibility curve always has a negative slope, it is likely that most spinwaves will propagate in a direction that has a large component perpendicular to the biasing field and this likelihood increases the greater the amplitude of the initial slope.

The experimental data on double-frequency generation corroborates the analysis of Schlömann. The normalized susceptibility curve of material Y has the slope that indicates that most spinwaves propagate substantially parallel to the biasing field. The magnetization vectors of these spinwaves therefore precess in circular orbits and they cannot contribute to frequency doubling. This explains why the double-frequency output of material Y saturates soon after the power level exceeds that at which the normalized susceptibility curve drops below 1. The normalized susceptibility curve of material S has the shape that indicates it is very likely that most spinwaves propagate in a direction that has a large component perpendicular to the biasing field. The magnetization vectors of most of these spinwaves precess in orbits whose ellipticities are comparable to those of a thin slab. This explains why there is no saturation in the double-frequency output, even though spinwaves have drastically reduced its suceptibility.

Eq. (1) predicts a constant slope of 2.0 for the curve of double-frequency output power vs input power, when powers are expressed in decibels. Even though (1) was derived for a very thin slab, the prediction of a slope of 2.0 is not altered by the use of thick slabs. However, all planar ferrites tested exhibited a slope substantially greater than 2.0 under certain conditions. It was most prominent in the relatively thin slabs (0.03 in or less). The typical pattern was as follows. The slope was 2.0 at low input powers. Over a range of intermediate input powers, the slope exceeded 2.0. At higher powers, the slope reverted to 2.0. The maximum slope observed was 2.6. The total increase in double-frequency output power due to the slope being greater than 2.0 over a range of input power varied from 3–7 db in the

[10] E. Schlöman, "Ferromagnetic Resonance at High Power Levels," Raytheon Co., Waltham, Mass., Rept. No. R-48; October 1, 1959.

[11] E. Schlömann, J. J. Green, and U. Milano, "Recent developments in ferromagnetic resonance at high power levels," *J. Appl. Phys.*, suppl. to vol. 31, pp. 386S-395S; May, 1960.

ferrites tested. At slab thicknesses where the maximum conversion gain occurred, and at greater thickness, the slope did not appreciably exceed 2.0. None of the isotropic ferrites tested showed a slope greater than 2.0.

The reason the slope is greater than 2 for planar ferrites is not clearly understood. However, the fact that it occurred for planar ferrites and not for isotropic ferrites makes it seem likely that spinwaves are responsible. In planar ferrites, the magnetization vectors of all spinwaves precess in highly elliptical orbits and are therefore very effective in frequency doubling. However, in isotropic ferrites, the ellipticities of the orbits depend strongly on the direction of propagation of the spinwaves and some of the spinwaves excited will be ineffective or only moderately effective in frequency doubling.

At slab thicknesses where the maximum gain occurs, and at power levels high enough to excite spinwaves, the amplitude of the double-frequency RF magnetic field in the ferrite will be very large and may, in fact, be comparable to or greater than the primary frequency field. This will change the spinwave excitation picture, and a behavior pattern that occurred at smaller thicknesses will not necessarily occur at the larger thicknesses. In particular, it should be noted that the double-frequency RF magnetic field may itself excite spinwaves through parallel pumping. The above explains why the slope of output vs input curve did not appreciably exceed 2.0 at the relatively larger thicknesses.

Maximum Conversion Efficiency

At an input peak power level of 6300 w, the maximum conversion efficiency obtained with a straight ferrite slab was −11.2 db; this was obtained with material P3 in a thickness of 0.0375 in. A conversion efficiency of −11.1 db was obtained with a planar sample 0.036 in thick, dielectrically loaded with a 0.0115-in slab of alumina. The ferrite was of the same chemical composition as P3 and P4 but fired under slightly different conditions.

Conclusions

The variation of the double-frequency conversion efficiency of a ferrite slab as a function of slab thickness can be explained by a surface-wave effect that tends to concentrate the double-frequency fields in the vicinity of the ferrite. Calculated values are in reasonable agreement with experimental results. The effect of spinwaves on frequency doubling can be understood on the basis of the ellipticity of the precession orbits of their magnetization vectors. Planar ferrites have an advantage over isotropic ferrites in frequency doubling, since a large planar anisotropy field causes large ellipticities in the precession orbits of both the uniform precession and the spinwaves. The relatively large dielectric constant of the planar ferrites is also helpful in frequency doubling.

Appendix

Derivation of (1)

Equation for Double-Frequency Magnetization, m_y

$$m_y(x) = AM_s\left[\frac{h_x(x)}{\Delta H}\right]^2 \tag{12}$$

$$A = \frac{1}{2}\,\frac{1+\sqrt{1+\left(\frac{2H_i}{\delta}\right)^2}}{1+\left(\frac{2H_i}{\delta}\right)^2}$$

$$\delta = H_a + 4\pi M_s(N_z - N_x)$$

$$H_i = \omega/\gamma.$$

Equation for Magnetic Field at Narrow Wall of Waveguide (TE_{10} Mode)

$$h_x(x) = h_x(0)e^{-\alpha x} \tag{13}$$

$$h_x(0) = \left[\frac{P_\omega \pi}{cd\sqrt{\mu_0/\epsilon_0}}\cdot\frac{\lambda_g\lambda_0}{\lambda_c^2}\right]^{1/2}. \tag{14}$$

Equation for Attenuation Factor

$$\alpha = \frac{ab}{cd}\,\frac{2\pi}{\lambda_c^2}\,\chi''. \tag{15}$$

χ'' is the susceptibility of the ferrite at ferromagnetic resonance. This equation can be inferred from (9.55) of Soohoo's book.[12]

$$\chi'' = \frac{4\pi M_s}{\Delta H}\,D \tag{16}$$

where

$$D = \frac{1+\sqrt{1+\left(\frac{2H_i}{\delta}\right)^2}}{\sqrt{1+\left(\frac{2H_i}{\delta}\right)^2}}.$$

Eq. (15) was derived in a recent paper by Bady.[13]

Equation for Magnetic Dipole Moment

From the above equations, we can write the equation for magnetization as

$$m_y(x) = \frac{AM_s}{(\Delta H)^2}\cdot\frac{P_\omega\pi}{\sqrt{\mu_0/\epsilon_0}}\,\frac{\lambda_g\lambda_0}{\lambda_c^2}\,\frac{e^{-2\alpha x}}{cd}$$
$$= m_y(0)e^{-2\alpha x}. \tag{17}$$

In view of assumption 4), we can obtain the total ef-

[12] R. F. Soohoo, "Theory and Application of Ferrites," Prentice-Hall, Inc., Englewood Cliffs, N. J.; 1960.

[13] I. Bady, "Planar ferrites at microwave frequencies," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 52–62; January, 1961.

fective dipole moment m_t by the integration

$$m_t = \int m_y dV = ab \int_0^\infty m_y(0) e^{-2\alpha x} dx$$

$$= \frac{A}{D} \frac{P_\omega \lambda_0}{16\pi\sqrt{\mu_0/\epsilon_0}\Delta H} \tag{18}$$

Equation for Double-Frequency Magnetic Field

$$h_{2\omega} = (4\pi)^2 10^{-9} \frac{2\omega m_t}{\sqrt{\mu_0/\epsilon_0}cd} \cdot \frac{\lambda_0}{\lambda_g} \cdot \tag{19}$$

This equation can be inferred from Goubau.[6]

Equation for Double-Frequency Power

$$P_{2\omega} = h_{2\omega}^2 \frac{cd\sqrt{\mu_0/\epsilon_0}}{\pi} \frac{\lambda_g}{\lambda_0} \cdot \tag{20}$$

The above equations will give the instantaneous peak power. Multiplying the right-hand side by 2 to get rms peak power, and combining with previous equations, (1) is obtained.

Comments on Model Used to Calculate Surface-Wave Effect

One of the ways in which the model differs from the actual geometry is that the model does not reflect the presence of the waveguide dimension d. In the model, the ratio between the phase constant when the ferrite slab is very thick and the phase constant when the slab is very thin approaches the square root of the dielectric constant. However, in the actual geometry, the ratio is greater by the factor λ_g/λ_0. As a result, the factor D [see (9)] at a given thickness will be smaller for the model than it is in the actual geometry. This causes the theoretically calculated over-all gain to be too high.

Another way in which the model differs from the actual geometry is that the model is unlimited in the Y direction, whereas in the actual geometry, the ferrite slab has a finite height. One important consequence is that in the actual geometry the surface wave effect can reduce the effective extent of the RF fields in the Y direction as well as in the Z direction; in the model, this is possible only in the Z direction. This means that the factor G, and therefore the theoretically calculated over-all gain for the model will be too low. Considering that the ratio d/b is equal to 4, and that the RF double-frequency fields in the Y direction in the actual geometry vary according to a cosine law, the maximum discrepancy between the model and the actual geometry due to this factor will be 3 db.

Another consequence of the fact that the actual ferrite slab is finite in the Y direction is that the ratio of double-frequency energy in the slab to energy in the air will be greater in the model than in the actual geometry, for a given slab thickness. As a result, the concentration of energy in the vicinity of the slab in the actual geometry will be smaller than in the model. This explains why the theoretically calculated thicknesses at which maximum over-all gain occurred were consistently lower than the measured thicknesses.

Consideration was given as to whether k or β_2 would be more appropriate to use as the multiplier of the term $(R-1)$ in (9). k is the phase constant of model in the case of a vanishingly thin slab. However, β_2 is the phase constant of the actual geometry in the case of a vanishingly thin slab, and its use was deemed more appropriate.

If k had been used, the maximum theoretical gain would be smaller and the theoretical thickness at which the maximum over-all gain occurred would shift slightly to a lower value, as compared to calculations made with the use of β_2. For example, with $\epsilon_r = 19$, and the attenuation at 0.04 in equal to 10 db/in, the maximum over-all gain would be smaller by 2.5 db; for an attenuation of 6 db/in, the reduction in maximum over-all gain would be 3.5 db. The shift in the thickness at which maximum over-all gain occurred would be about 2 per cent.

Reasonable agreement would be obtained between calculated results and experimental results regardless of whether β_2 or k were used. However, better agreement was obtained with the use of β_2. Apparently, with the use of β_2, there was a good deal of mutual cancellation among the different effects due to the difference between the model and the actual geometry.

It has been assumed in this paper that all of the double-frequency energy couples completely from the surface mode to the rectangular waveguide TE_{01} mode. It should be noted that if the double-frequency waves were directed into the waveguide section containing the ferrite slab from an outside source, the ferrite slab would be a small perturbation and the fields would couple only weakly to the surface mode. However, when the source of the double-frequency energy is in the ferrite itself, the surface mode is the principal mode.

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